

लघु 3-तरीज प्रश्न

1. $\sin 75^\circ$ का मान ज्ञात करें।

$$\sin 75^\circ$$

$$= \sin (45^\circ + 30^\circ) \left[\because \sin(A+B) = \sin A \cdot \cos B + \cos A \cdot \sin B \right]$$

$$= \sin 45^\circ \cdot \cos 30^\circ + \cos 45^\circ \cdot \sin 30^\circ$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}} \text{ AB}$$

2. एल करें : $\sin x = -\frac{\sqrt{3}}{2}$

$$\therefore \sin x = -\frac{\sqrt{3}}{2} \left[x \in [\pi, 2\pi] \right]$$

$$\sin x = -\sin \left(\frac{\pi}{3} \right)$$

$$\sin x = \sin \left(\pi + \frac{\pi}{3} \right)$$

$$x = \pi + \frac{\pi}{3}$$

$$\text{III } x = \frac{4\pi}{3} \text{ AB } \left[\because \sin x = -\frac{\sqrt{3}}{2}, \text{ III, IV} \right]$$

$$\text{IV } x = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3} \text{ AB}$$

(3) समीकरण $\tan x = \sqrt{3}$ को हल करें,
 $\therefore \tan x = \sqrt{3}, \quad 0 \leq x < 2\pi$

I $\tan x = \sqrt{3}$

$$\tan x = \tan \frac{\pi}{3}$$

$$\boxed{\therefore x = \frac{\pi}{3}} \text{ Ans}$$

III $x = \pi + \frac{\pi}{3}$

$$\boxed{\therefore x = \frac{4\pi}{3}} \text{ Ans}$$

(4) सम्मिश्र संख्या को $a+ib$ के रूप में व्यक्त करें।
 $\rightarrow 3(7+i7) + i(7+i7)$

$$\therefore 3(7+i7) + i(7+i7)$$

$$= 21 + 21i + 7i + 7i^2$$

$$= 21 + 28i + 7 \times -1$$

$$= 21 + 28i - 7$$

$$= 14 + 28i \text{ , Ans}$$

(5) $\sqrt{5}-3j$ का वहुपात्मक प्रतिनिधि ज्ञात करें।

$$\therefore \text{वहुपात्मक प्रतिनिधि} = \frac{1}{\sqrt{5}-3j} \times \frac{\sqrt{5}+3j}{\sqrt{5}+3j}$$

$$= \frac{\sqrt{5}+3j}{5-9j^2} = \frac{\sqrt{5}+3j}{5-9(-1)}$$

$$= \frac{\sqrt{5}+3j}{14} = \frac{\sqrt{5}}{14} + \frac{3}{14}j \quad \text{Ans}$$

7) यदि $z_1 = 2-j$ तथा $z_2 = -2+j$ तो z_1+z_2 का माप ज्ञात करें।

$$\therefore z_1+z_2 = (2-j) + (-2+j)$$

$$= 2-j-j = 2-2j$$

$$= 0$$

$$\therefore |z_1+z_2| = |0|$$

$$= 0 \quad \text{Ans}$$

7. सिद्ध करें कि $1 + i^2 + i^4 + i^6 = 0$

$$\text{L.H.S} = 1 + i^2 + i^4 + i^6$$

$$= 1 + (-1) + 1 + (-1)$$

$$= 1 - 1 + 1 - 1$$

$$= 0 \text{ R.H.S proved}$$

8. ^{9.} हल करें : $3x^2 + 15 = 0$

$$\therefore 3x^2 + 15 = 0$$

$$3x^2 = -15$$

$$x^2 = -5$$

$$\therefore x = \pm \sqrt{-5}$$

$$x = \pm \sqrt{-1 \times 5}$$

$$x = \pm \sqrt{i^2 \times 5}$$

$$x = \pm i\sqrt{5} \text{ Ans}$$

9. समिश्र संख्या $(2-5j)^2$ की संयुग्मी ज्ञात करें।

$$\therefore (2-5j)^2$$

$$= (2)^2 + (5j)^2 - 2 \cdot 2 \cdot 5j$$

$$= 4 + 25j^2 - 20j$$

$$= 4 + 25(-1) - 20j$$

$$= 4 - 25 - 20j$$

$$= -21 - 20j$$

$$\therefore \text{संयुग्मी} = -21 + 20j \quad \text{Ans}$$

10. हल करें: $\frac{5-2x}{3} \leq \frac{x}{6} - 5$

$$\therefore \frac{5-2x}{3} \leq \frac{x}{6} - 5$$

$$\frac{5-2x}{3} < \frac{x-30}{2}$$

$$10 - 4x < x - 30$$

$$10 + 30 < x + 4x$$

$$84 < 5x$$

$$8 < x$$

$$\therefore x \geq 8$$

$$x = 8, 9, 10, 11, \dots \quad \text{Ans}$$

11. हल करें: $5x - 7 < 3x + 1$

$$\therefore 5x - 7 < 3x + 1$$

$$5x - 3x < 1 + 7$$

$$2x < 8$$

$$x < 4$$

$$\therefore x = \dots, -1, 0, 1, 2, 3$$

12. यदि $\frac{nP_4}{n-1P_4} = \frac{5}{3}$, तो n का मान ज्ञात करें।

$$\frac{n(n-1)(n-2)(n-3)}{(n-1)(n-2)(n-3)(n-4)} = \frac{5}{3}$$

$$\frac{n}{n-4} = \frac{5}{3}$$

$$3n = 5n - 20$$

$$3n - 5n = -20$$

$$-2n = -20$$

$$\therefore n = 10$$

13) यदि $nC_{10} = nC_4$ तो n का मान ज्ञात करें।

$$n - 10 = 4$$

$$n = 4 + 10$$

$$\therefore n = 14$$

$${}^{14}C_4 = 1001 \text{ Ans}$$

14) यदि $\frac{1}{6} + \frac{1}{7} = \frac{x}{8}$, x का मान ज्ञात करें।

$$\therefore \frac{1}{6} + \frac{1}{7} = \frac{x}{8}$$

$$\frac{7+6}{42} = \frac{x}{8}$$

$$\frac{13}{42} = \frac{x}{8}$$

$$42x = 104$$

$$\therefore x = \frac{104}{42}$$

$$= \frac{52}{21}$$

$$\therefore x = \frac{52}{21} \text{ Ans}$$

15) यदि $\frac{1}{66} + \frac{1}{77} = \frac{x}{8}$, x का मान ज्ञात करें।

$$\therefore \frac{1}{66} + \frac{1}{77} = \frac{x}{8}$$

$$\frac{7+6}{462} = \frac{x}{8}$$

$$\frac{13}{462} = \frac{x}{8}$$

$$100x = 4692$$

$$x = \frac{4692}{100}$$

$$= \frac{4692}{100} = 46.92$$

$$\therefore x = \frac{52}{231}$$

दीर्घ उत्तरीय प्रश्न

(Q.6) सिद्ध करें कि $\cos^2 x + \cos^2(x + \frac{\pi}{3}) + \cos^2(x - \frac{\pi}{3}) = \frac{3}{2}$

$$\text{L.H.S} = \cos^2 x + \cos^2(x + \frac{\pi}{3}) + \cos^2(x - \frac{\pi}{3})$$

$$= \because \cos^2 \theta = \frac{1 + \cos 2\theta}{2}$$

$$= \frac{1}{2} \left[(1 + \cos 2x) + (1 + \cos(2x + \frac{2\pi}{3})) + (1 + \cos(2x - \frac{2\pi}{3})) \right]$$

$$= \frac{1}{2} \left[3 + \cos 2x + \cos(2x + \frac{2\pi}{3}) + \cos(2x - \frac{2\pi}{3}) \right]$$

$$\because \cos A + \cos(A+B) + \cos(A-B) = 0 \left(\because B = \frac{2\pi}{3} \right)$$

$$\therefore \frac{1}{2} \times 3 = \frac{3}{2} \text{ R.H.S}$$

proved

27. समिन् सरेला $-4 + j4\sqrt{3}$ का ध्रुवीय रूप में लकी।

$$\therefore z = -4 + j4\sqrt{3}$$

$$\text{मापोक} = r = \sqrt{(-4)^2 + (4\sqrt{3})^2} = \sqrt{16 + 48} = 8$$

$$\therefore \tan \theta = \frac{4\sqrt{3}}{-4} = -\sqrt{3}$$

$$\theta = \frac{2\pi}{3}$$

$$\therefore \text{ध्रुवीय रूप} : z = 8 \left(\cos \frac{2\pi}{3} + j \sin \frac{2\pi}{3} \right) \quad \text{Ans}$$

28. द्विघात समीकरण $x^2 + x + \frac{1}{\sqrt{2}} = 0$ का हल करें।

$$\sqrt{2}x^2 + \sqrt{2}x + 1 = 0$$

$$a = \sqrt{2}$$

$$b = \sqrt{2}$$

$$c = 1$$

$$\therefore D = b^2 - 4ac$$

$$= (\sqrt{2})^2 - 4 \times \sqrt{2} \times 1$$

$$= 2 - 4\sqrt{2} = 2(1 - 2\sqrt{2})$$

$$\therefore x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-\sqrt{2} \pm \sqrt{2(1 - 2\sqrt{2})}}{2 \times \sqrt{2}} = \frac{-\sqrt{2}(-1 \pm \sqrt{1 - 2\sqrt{2}})}{2 \times \sqrt{2}}$$

$$\therefore x = \frac{-1 \pm \sqrt{1 - 2\sqrt{2}}}{2}$$

30. यदि $5 \times {}^4P_x = 6 \times {}^5P_{x-1}$ तो x तथा 8P_x का मान ज्ञात करें।

हल: $5(4!) = \frac{6(5!)}{(6-x)!} \div \frac{4!}{(5-x)!}$

$$1 \cdot 5 = \frac{6 \times 5}{6-x}$$

$$6-x = 6$$

$$\boxed{\therefore x = 0} \quad \text{Ans}$$

तब

$${}^8P_x = {}^8P_0$$

$$= 1 \quad \text{Ans}$$